COMPOSING WITH SIEVES:

STRUCTURE AND INDETERMINACY *IN-TIME*.

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ABSTRACT

Introduced by Xenakis 50 years ago, sieves have proved to be a relevant and robust device for music composition. Examples of complex and symmetric sieves usage in original works are presented along with a few possible applications not explored before. The dichotomy between predetermined abstract structures such as sieves and their actualization through random procedures is discussed and it is also shown that in the hands of a innovative musician, sieves not only serve the craft aspect of composing but could also reveal as well as impact deeper levels of thinking.

1. PERIODICITY, WEIGHTS, AND SYMMETRY.

Sieves were introduced during the early 1960s by Xenakis in his works but remained a rather esoteric topic until rather recently when a number of writings on the subject have appeared - Ariza [1], Gibson [6], Exarchos and Jones [5], Solomos [7] to name some - testifying to the relevance of this device for music composition. Since many basic aspects and in particular sieve analysis and construction have been discussed previously, only a few brief reminders are necessary here.

Sieves are logical filters expressed as boolean operations on congruence modulo classes. A trivial case is that of a sieve containing equivalence classes denoted by various indices (following the notation used by Xenakis) of a single modulo:

$$13_0 \text{ U } 13_3 \text{ U } 13_5 \text{ U } 13_8. \qquad (1)$$

This formula will generate a periodic sequence of numbers with 13 the only modulo able to define elements of the sieve since it is a prime number. Messiaen's modes with limited transpositions can be generated with simple periodic sieves:

$$3_0 \text{ U } 3_1 \text{ (second mode, an octatonic scale)} \qquad (2)$$

while the expression offered by Xenakis in *Formalized Music* [13] for generating the major scale contains two modulo terms and a more involved set of operations:

$$(\overline{3}_{n+2} \cap 4_n)\text{U}(\overline{3}_{n+1} \cap 4_{n+1})\text{U}(3_{n+2} \cap 4_{n+2})\text{U}(\overline{3}_n \cap 4_{n+3}) \qquad (3)$$

In this case, the period of the sieve is the lowest common multiple (LCM) of its modulo terms while the indices show the possibility of transposing the scale. Similar pursuits, albeit from a different perspective, may be found in Anatol Vieru's *Book of Modes* [12]. Xenakis favored "aperiodic" pitch sieves - actually, sieves with a period longer than the actual range of the sound source, hence making it impossible to determine its period. There is a clear and desirable distinction between such a pitch sieve and any octaviating scales, tonal or atonal. On the other hand, oscillating in the same piece between periodic and "aperiodic" sieves, between a recognizable structure and apparent disorder offers an enticing way of organizing musical materials.

Multiple modulo terms and more intricate boolean operations allow for the addition of another feature: weights establishing preferences among the elements of the sieve. Weights may be assigned to individual elements in order to establish a hierarchy and to transform a "scale" - a list - into a "mode". In DISSCO, a Digital Instrument for Sound Synthesis and Composition developed at University of Illinois Computer Music Project and Argonne National Laboratory [2], weights could follow a pattern in sync with that of the sieve elements or they could have an independent cycle or no cycle at all (aperiodic). In turn, each cycle could have its own scaling factor: e.g., the mid range octaves could have more sway than the extreme ones. Although the weights may be assigned arbitrarily following the judgement of the composer, a more abstract arrangement assigns a particular weight to each module thus reinforcing the internal structure of the sieve:

sieve:	$(3_0 \cap 4_0)$ U	$3_1 \cap$	$(4_0$ U	$4_3)$ U	$3_2 \cap$	$(4_1$ U	$4_3)$	
modulo weight:	15	1	10	1	1	5	1	1
resulting elements:	{0}		{4, 7}			{5, 11}		
element weight:	16		12 each			7 each		

Figure 1. Modulo weights

Applied to diatonic pitches, the results in this example would favor the tonic triad {0, 4, 7} over the dominant tritone {5, 11}.

Sieves that produce symmetric intervals between numbers contain modulo terms that have symmetric indices. A simple nonretrogradable rhythm:

attacks	0	1	9	11	19	20
durations	1	8	2	8	1	

Figure 2. Nonretogradable rhythm

will be produced by the sieve:

$$(5_0 \cap 4_0) \cup (5_1 \cup 5_4) \cap (4_1 \cup 4_3) \quad (4)$$

where the terms 5_1, 5_4 and 4_1, 4_3 are symmetric with respect to the origins 5_0 and 4_0 since the sieve is periodic and $5_0 \equiv 5_5$. The sieve generating the Dorian mode shows a similar balance:

$$(3_0 \cap 4_0) \cup 3_1 \cap (4_2 \cup 4_3) \cup 3_2 \cap (4_2 \cup 4_1) \quad (5)$$

An interesting case is that of rhythmic sieves that are symmetric and extended over very large areas of a piece; they can be used to create structures similar to that of the first movement of the *Symphony Op. 21* by Webern or Machaut's *Ma fin est mon commencement*.

Multiple-entry sieves, an even more elaborate construct, could be described as involving conditional probability, or as multidimensional matrices (not unlike the sequence of "screens" used by Xenakis to generate *Analogique A et B*) or as related to the more common sieves through the use of equivalence modulo m relations of the type:

$$\mathbf{k_1 m_1} + \mathbf{k_2 m_2} + \ldots + \mathbf{k_i m_i} + \mathbf{n} \quad (6)$$

This expression is helpful, for instance, when determining the position of an attack measured in the smallest time quanta available. With $\mathbf{k_1}$ being the measure number, $\mathbf{k_2}$ the number of a beat in a 3/4 measure and the sixteen the smallest duration item or the EDU (Elementary Displacement Unit in the terminology introduced by Xenakis),

$\mathbf{m_2}$ = 4 sixteenths,
$\mathbf{m_1}$ = 3 beats $\cdot$ $\mathbf{m_2}$ = 12, and
$\mathbf{n}$ = number of a particular sixteen in a beat.

The third sixteen of the second beat in the 7th measure will then be:

$$6 \cdot 12 + 1 \cdot 4 + 2 = 78 \quad (7)$$

(first member of each group is always 0). If instead we consider

$\boldsymbol{m_2}$ the number of all dynamic levels in a piece,
$\boldsymbol{m_1}$ the number of all available pitches $\cdot$ $\mathbf{m_2}$,
$\mathbf{n}$ a particular dynamic level,
$\mathbf{k_2}$ a particular pitch, and
$\mathbf{k_1}$ a particular instrument in a group of size $\geq$ $\mathbf{k_1}$,

we can create orchestration constraints. The period of such a sieve will be $\boldsymbol{m_1}$ and its modulo numbers will have to be divisors of $\boldsymbol{m_1}$ $\cdot$ total number of instruments.

2. APPLICATIONS

A symmetric rhythmic sieve that includes over 100 modulo terms and operations was used to create an extended palindrome in *Cuniculi*, for five tubas [10]. The same symmetric sieve is repeated throughout the piece in an ostinato of palindromes that cover its entire 10:30 minutes duration:

|→→→:←←←|→→→:←←←| etc.

Figure 3. Rhythmic palindrome ostinato

The nonretrogradable rhythm forms a firm and elaborate scaffolding for the piece but its length and complexity make it difficult to be detected by the listener. In order to differentiate between the repeats of the nonretrogradable rhythm, the complexity of the pitch material, the dynamic levels and other parameter values are incremented constantly as we advance from left to right, along with the general entropy - the measure of disorder, that defines the passing of time.

The symmetry of the rhythm insures that one can proceed either forward or in reverse either from the beginning or from the end of the palindrome as well as from its central axis since the succession of time intervals is the same in both directions . It also allows skipping over one or more repeats of the sieve in either direction creating a "time machine" that may facilitate travels in the past (memory) or foreshadow the future. In the case of *Cuniculi*, the symmetry engenders a static object, associated with a cyclical time, the increase in entropy defines the arrow of time, a linear evolution, while the resulting music embodies the synthesis between the two opposing views contained in this dialectics.

It must be clear by now that sieves with hundreds of terms and complex structures can not be easily handled by hand. *Cuniculi* was realized with MP1 [9], an older program designed to generate compositions for voices and acoustic instruments; DISSCO, the software presently in use, integrates composition and sound synthesis bringing them together in a seamless process and has similar but enhanced capabilities.

A different instance in which an ostinato of symmetric rhythmic sieves proves to be useful is exemplified in a work in progress for violin and computer-generated sounds also composed with DISSCO. In it three types of textures ***A***, ***B***, ***C*** are permuted and associated, three at a time, with a sieve that has the following structure: $\boldsymbol{\alpha}\ \bar{\boldsymbol{\alpha}}\ \boldsymbol{\beta}\ \bar{\boldsymbol{\beta}}\ \boldsymbol{\alpha}\ \boldsymbol{\alpha}$ where a $\boldsymbol{\beta}$ segment is half the size of an $\boldsymbol{\alpha}$ segment and $\bar{\boldsymbol{\alpha}}$ $\bar{\boldsymbol{\beta}}$ are the retrogrades of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ respectively. The other sound parameters do not follow the retrograde patterns and present different materials for each sieve segment: $\boldsymbol{A_1}$ $\boldsymbol{A_2}$ $\boldsymbol{b_1}$ $\boldsymbol{b_2}$ $\boldsymbol{C_1}$ $\boldsymbol{C_2}$ each type of texture being connected to a forward-retrograde pair of sieve segments (ex. $\boldsymbol{A_1}$ $\boldsymbol{A_2}$ corresponding to $\boldsymbol{\alpha}\ \bar{\boldsymbol{\alpha}}$). Due to the symmetry of the rhythm, the following combinations are possible:

[$\boldsymbol{A_1}$ $\boldsymbol{A_2}$]; [$\boldsymbol{A_1}$ $\boldsymbol{A_1}$]; [$\boldsymbol{A_2}$ $\boldsymbol{A_2}$]; [$\boldsymbol{A_2}$ $\boldsymbol{A_1}$]

for the first texture,

[$\boldsymbol{b_1}$ $\boldsymbol{b_2}$]; [$\boldsymbol{b_1}$ $\boldsymbol{b_1}$]; [$\boldsymbol{b_2}$ $\boldsymbol{b_2}$]; [$\boldsymbol{b_2}$ $\boldsymbol{b_1}$]

for the middle texture, and

[$\boldsymbol{C_1}$ $\boldsymbol{C_2}$]; [$\boldsymbol{C_1}$ $\boldsymbol{C_1}$]; [$\boldsymbol{C_2}$ $\boldsymbol{C_2}$]; [$\boldsymbol{C_2}$ $\boldsymbol{C_1}$]

for the last texture. Moreover, a grouping of the three texture types {***ABC***} is sometimes coupled with more than one statement of the sieve and when one of these grouping encompasses two or three iterations of the rhythmic palindrome more choices become available.

The number of possible combinations for the **α α** portion of the sieve increases from 4 (for one iteration) to 9 (for two iterations) or 16 (for three iterations).

Next, probabilities are assigned to each of these pairs and, in the case of synthesized sounds, choices are made with the help of a random number generator. The stream of electro-acoustic sounds is computed in advance and a new variant of the piece is generated for each performance. For the violin, all alternatives are provided in the score and the performer is asked to choose one of them: the ostinato of rhythmic palindromes provides a foundation for areas of aleatory music while, at the same time, insuring the coherence and the integrity of the process. The live musician could choose what path to follow either ahead of time or, preferably, at the time of the performance. In the latter case, the choices made by the human artist on the spot will be influenced - if only subliminally - by the music coming out of the loudspeakers which, again, will be different every time.

Another rather straightforward application of sieves consists in establishing pitch areas easily identifiable and related to each other, then finding ways of "modulating" from one area to the next. This can be done by selecting subsets of a sieve with many elements and either starting with smaller subsets and progressively adding new sounds or by using intersections between subsets, in other words manipulating the modulo terms of the more complex sieve. Admittedly, this is not a procedure as elegant and sophisticated as the *metabolae* proposed by Xenakis [13] but it is effective nevertheless.

All procedures mentioned above represent features available in DISSCO and have been employed in actual compositions. A number of other sieve applications are candidates for becoming future components of the software. Weighted sieves could control the presence and the amplitude of sound partials in additive synthesis either by creating models of acoustic instrumental sounds or, in a more abstract way, by paralleling other proportions in the structure of the work; the resonance of instrument bodies could be simulated through complex weighted sieves; and formants, a combination of both resonance and control of partials, could be created. Since in DISSCO, analogous operations are present at all structural levels, one can also conceive formants at the macro level of the piece as envisaged by Stockhausen (*Gruppen*) or even as means of organizing the form of the piece as proposed by Boulez (*Third Piano Sonata*).

3. TEMPLATES AND REALIZATIONS.

In the hands of a diligent and creative composer, sieves could become a powerful tool. Their usefulness can also go beyond the craft aspect of composing and both reveal as well as impact deeper levels of thinking.

An obvious way to apply a sieve is to use all its elements in the same way every note of a scale is employed in a traditional piece; assigning weights to its components will enhance its internal organization without changing the way it functions. In the works mentioned above as well as in many other ones, sieves were treated as templates, as potentialities that might or might not be fully realized. In the time domain, for instance, a weighted sieve engendering a large number of attacks defines only places where sounds *could* occur without guaranteeing that any will be assigned at any particular location. In a computer-assisted composition, a linear density smaller than the density of the sieve coupled with selections based on random procedures will insure that not all possible locations will be activated. Similarly, a pitch sieve rich in available choices might not be fully utilized at the local level thus masking the extent of the pitch reservoir available for a larger area.

The play between structure and randomness, between determinism (sieve) and chance, mirrors the natural world where the laws of physics allow for more virtual events than actual happenings.

The predetermined structure (physical laws, sieve, etc.) can not be contradicted and its actualization has to conform. At the same time, these actual manifestations are unpredictable, the result of causal chains too complex to follow, of pure chance or, in our case, the result of applying stochastic distributions. This way, the music results from the meeting of the *possible* ascertained by the sieve/template and the *probable* represented by random procedures.

> There are many flowers
> But few will bear fruit;
> They all knock at life's gate
> Yet many blossoms die. [4]

Such ideas are implemented through DISSCO when producing *manifold compositions,* multiple variants of the same piece that emphasize the interaction between structure and indeterminacy [11]. They are *composition classes* generated by a computer that runs a program containing elements of indeterminacy and reads the same data for each variant. As members of an *equivalence class*, they share the same structure and are the result of the same process, but deviate in the way specific events are arranged in time. Similar to faces in a crowd, they have the same basic features but differ in details and reflect various personalities.

4. ABOUT TIME.

Another way of looking at the dichotomy between structure and indeterminacy is through the *outside-time*/*in-time* categories considered by Xenakis. Sieves are typical examples of *outside-time* abstract structures, preceding any attempts to fashion specific events, while the *in-time* product, the piece - a variant of the *manifold* in DISSCO's case, is the result of random processes being applied to abstract templates. Such *outside-time* "architectures" (as Xenakis calls them) are static, frozen in time and betray a modernist-structuralist way of thinking in the view of Jean-Jacques Nattiez [8] who points out that, by contrast, post-modernism is more concerned with the passing of time.

In fact, Xenakis also defines a third category, the *temporal* which might be puzzling until one realizes that although sound parameters have an Abelian (commutative) group structure, physical time, being irreversible, does not have a group structure (i.e. there are no inverses). The *temporal* category underlines this peculiarity of the experiential time that sets it apart from the other aspects of a musical composition. Now, although an asymmetric succession of arbitrary durations does not form a group, nonretrogradable rhythms and iterations such as ostinatos or traditional, constant steady meter, do. They create a cyclical time associated with the "eternal return" identified by Mircea Eliade as characteristic to all rituals [3]; they compel us to participate in an exemplary action that is repeated at regular intervals and supersede our perception of the mundane flowing of "ordinary" time.

In DISSCO, indeterminate processes that distribute events *in-time* complement or even upset these unhistorical, cyclical returns created by sieves templates. Since a variant of the *manifold* is generated in one seamless run, it owes its distinct integrity, its own "personality", to a particular "history" provided by a sequence of random numbers. However, a random number generator actually produces **pseudo** random numbers and the same seed engenders the same exact output. An interesting situation is then created: the events in the piece depend on indeterminacy but the sequence of random numbers, the source of randomness, is itself a deterministic chain. One more layer in the play between structure and chance, causality and indeterminacy.

Sieves that generate rhythmic palindrome ostinatos also provide a gateway for virtual incursions in time and collections of relative, equal, but mutually exclusive aleatory-type alternatives. By combining the use of group structures with indeterminate procedures in the selection of some but not all possible time events, two worlds are made to compete with each other in complex and realistic ways. When the artistic product is also a *manifold composition* with its arbitrary number of actual and potential variants having the same *outside-time* structure, the relativity and the ephemerality of all *in-time* artifacts is underscored.

5. REFERENCES

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